**Tutorial Activity 7**

**Week 8**

In this tutorial, the objectives are as follows:

* To compute volatility forecasts using the conditional expectation from the theoretical model.
* To practise how to generate the return and volatility forecasts using R.
* To discuss the terms of ‘conditional variance’ and ‘unconditional variance’.

**Recall:**

1. Consider the following simple GARCH(1,1) model

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Suppose that the researcher had estimated the above GARCH model for a series of log returns on palm oil prices and obtained the following parameter estimates: ; , , , and . The researcher has data available up to and including time T=763.

1. Write down a set of equations which could be employed to produce one-, two-, and three-step-ahead forecasts for the conditional variance of .

We know all information including that available up to time *T*. The answer to this question will use the convention from the GARCH modelling literature to denote the conditional variance by rather than . What we want to generate are forecasts of ⏐ ΩT, ⏐Ω*T*, ..., ⏐Ω*T* where Ω*T* denotes all information available up to and including observation T. Adding 1 then 2 then 3 to each of the time subscripts, we have the conditional variance equations for times *T*+1, *T*+2, and *T*+3:

(1)

(2)

(3)

Let be the one step ahead forecast for made at time . This is easy to calculate since, at time , we know the values of all the terms on the RHS. Given , how do we calculate , that is the 2-step ahead forecast for made at time ?

From (2), we can write

(4)

where is the expectation, made at time , of , which is the squared disturbance term. The model assumes that the series has zero mean, so we can now write

The conditional variance of is , so

=

Turning this argument around, and applying it to the problem that we have,

but we do not know , so we replace it with , so that (4) becomes

What about the 3-step ahead forecast?

By similar arguments,

And so on. This is the method we could use to forecast the conditional variance of . If were, say, daily returns on the FTSE, we could use these volatility forecasts as an input in the Black Scholes equation to help determine the appropriate price of FTSE index options.

1. Compute the values of five point forecasts of conditional variance of .

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**Note:** the last five observations are omitted in the estimations for forecasting excersise. Total number of available observations is 763. Only 758 observations are used out of available 763 observations in the GARCH estimation.

**Table 1:** Last 3 values of log returns, residuals, and conditional standard deviation series after the estimation using 758 observations.

|  |  |  |  |
| --- | --- | --- | --- |
| **Observations** | **Returns** | **Residuals** | **Conditional standard deviation** |
| 756 | -0.175419872 | -0.1907061830 | 0.4430412 |
| 757 | -0.418491763 | -0.4359749185 | 0.4248941 |
| 758 | 0.180957995 | 0.16824626660 | 0.4357224 |

1. Suppose now that the coefficient estimate of for this model is 0.95 instead. By reconsidering the forecast expressions you derived in **part (a)**, explain what would happen to the forecasts in this case.

An *s*-step ahead forecast for the conditional variance could be written

|  |  |
| --- | --- |
|  | (5) |

For the new value of β, the persistence of shocks to the conditional variance, given by (*α*1+*β*1) is 0.1381+ 0.95 = 1.0881, which is bigger than 1. It is obvious from equation (5), that any value for (*α*1+*β*1) bigger than one will lead the forecasts to explode. The forecasts will keep on increasing and will tend to infinity as the forecast horizon increases (i.e. as *s* increases). This is obviously an undesirable property of a forecasting model! This is called “non-stationarity in variance”.

For (*α*1+*β*1) <1, the forecasts will converge on the unconditional variance as the forecast horizon increases. For (*α*1+*β*1) = 1, known as “integrated GARCH” or IGARCH, there is a unit root in the conditional variance, and the forecasts will stay constant as the forecast horizon increases.

1. Demonstrate volatility forecasting in R.
2. Distinguish between the terms ‘conditional variance’ and ‘unconditional variance’. Which of the two is more likely to be relevant for producing:
3. one-step-ahead volatility forecasts.
4. twenty-step-ahead volatility forecasts.

The unconditional variance of a random variable could be thought of, abusing the terminology somewhat, as the variance without reference to a time index, or rather the variance of the data taken as a whole, without conditioning on a particular information set. The conditional variance, on the other hand, is the variance of a random variable at a particular point in time, conditional upon a particular information set. The variance of , , conditional upon its previous values, may be written , while the unconditional variance would simply be .

Forecasts from models such as GARCH would be conditional forecasts, produced for a particular point in time, while historical volatility is an unconditional measure that would generate unconditional forecasts. For producing 1-step ahead forecasts, it is likely that a conditional model making use of recent relevant information will provide more accurate forecasts (although whether it would in any particular application is an empirical question). As the forecast horizon increases, however, a GARCH model that is “stationary in variance” will yield forecasts that converge upon the long-term average (historical) volatility. By the time we reach 20-steps ahead, the GARCH forecast is likely to be very close to the unconditional variance so that there is little gain likely from using GARCH models for forecasts with very long horizons.